Galactic Rotation Without Dark Matter Halos

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Introduction:

A central issue in the field of astrophysics is explaining why observed galactic rotation curves are essentially flat. In 1933 Fritz Zwicky of Caltech applied the virial theorem (shown below) to the Coma cluster of galaxies. His observations indicated that there must be much more matter present in the galaxies than what we observe (Fig. 1). If this “dark matter” did not exist, rotating galaxies would fly apart due to the forces incurred by rotation.

$$P.E.(system) \simeq -\frac{1}{2}G\frac{N^2m^2}{R_{tot}} = -\frac{1}{2}G\frac{M_{tot}^2}{R_{tot}}$$

$$\frac{1}{2}M_{tot}v^2 = +\frac{1}{4}G\frac{M_{tot}^2}{R_{tot}}$$

$$M_{tot} \simeq 2\frac{R_{tot}v^2}{G}$$

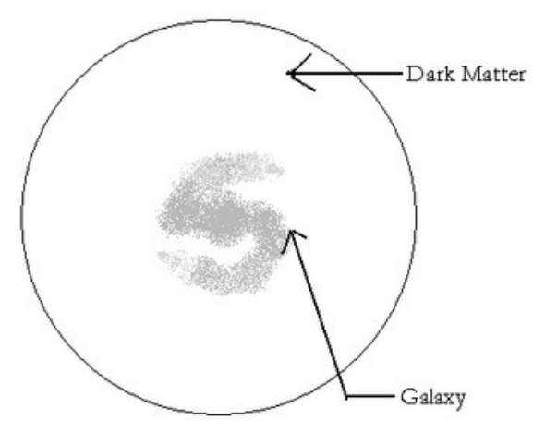

Fig. 1 – A dark matter halo surrounding a rotating galaxy

In addition to satisfying the virial theorem, dark matter also accounts for the problem of flat galactic rotation curves. Newtonian mechanics predicts that, for rotating galaxies, rotational velocity should decrease as distance from the axis of rotation increases. However, observations show that galactic rotation curves remain essentially flat in a manner similar to rigid body rotation. Our galaxy, the milky way, is an example of such a galaxy with a flat rotation curve as seen in Fig. 2.

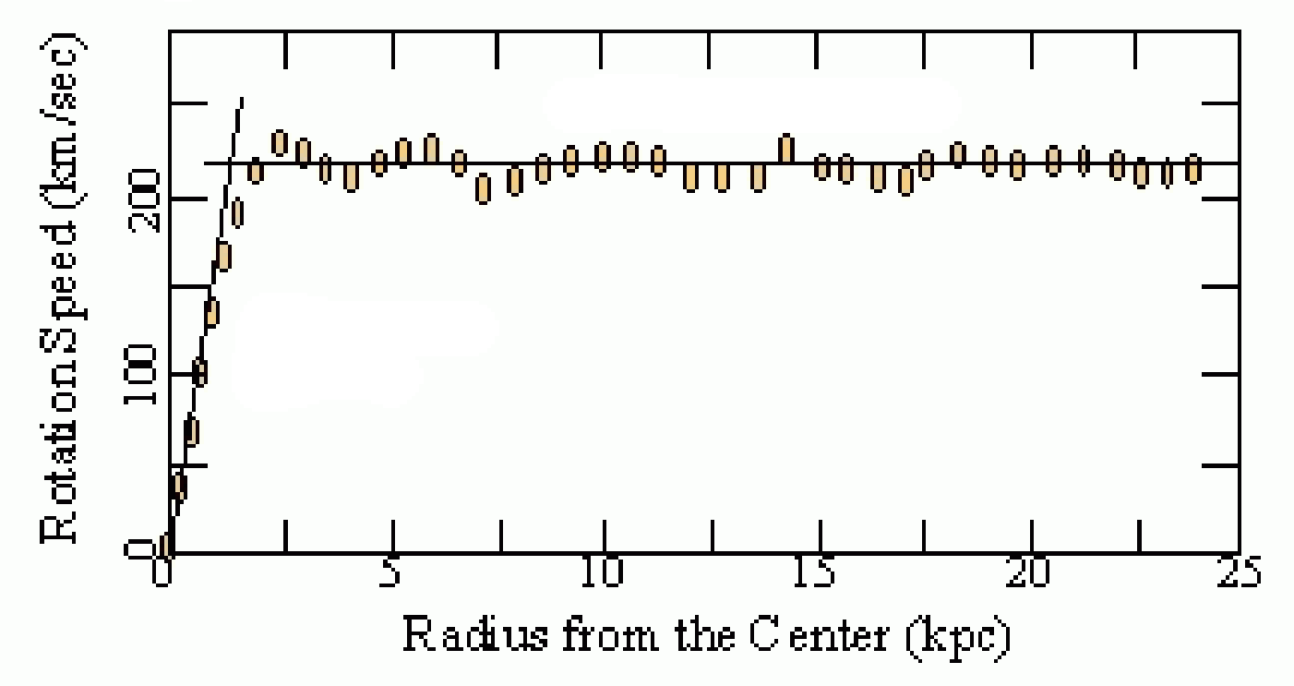

Fig. 2 – Galactic rotation curve for the Milky Way galaxy

Many theories have been developed in an attempt to account for the observations, but they are typically met with much skepticism as they are developed specifically to explain the phenomenon. General Relativity (GR) remains the preferred theory of gravity, with Newtonian theory as its limit. However, because the galactic field is weak (with the exception of the cores where black holes are said to reside) and the motions are non-relativistic (v << c) the conventional approach to studying galactic dynamics employs Newtonian gravity.

In July 2005, Copperstock and Tieu published a model of galactic rotation, using only General Relativity, that aimed to explain the observed rotation curves as correct in a general relativistic model and therefore negate the need for dark matter to explain the anomalies. They argue that because stars are gravitationally bound and that the problem of gravitationally bound objects in general relativity is non-linear, the Newtonian-based approach is inadequate. Our goal is to examine the Copperstock-Tieu model to determine its feasibility

The Cooperstock-Tieu Model

Within the context of General Relativity, it is common to model a galaxy as a uniformly rotating fluid without pressure and symmetric about its axis of rotation. The axially symmetric metric used in the CT model can be described with sufficient generality in the following form:

$$ds^2 = -e^{v-w}(udz^2+dr^2) - r^2e^{-w}d\phi^2 + e^w(cdt - Nd\phi)^2$$

Where u, v, w and N are functions of cylindrical polar coordinates r, z. It can be shown that to the order required, u can be taken to be unity. In addition, the simplest reference frame to work in is the one that is co-moving with the matter $U^i = \delta_0^i$ where U^i is the 4-velocity.

Mestel [5] considered a special rotating disk with surface density inversely proportional to radius. Using a disk potential with Bessel functions, he found that it can lead to an absolutely flat galactic velocity rotation curve. Cooperstock and Tieu build on this idea and derive equations for the disk potential Φ and the tangential velocity V

$$\Phi = \sum_{n=1} C_n e^{-k_n|z|} J_0(k_n r)$$

$$V = -c\sum_{n=1} k_n C_n e^{-k_n|z|} J_1(k_n r)$$

Where C_n is an arbitrary constant and k chosen so that the $J_0(k_n r)$ terms are orthogonal to each other. The curve fit below illustrates the accuracy of these expressions for the Milky Way (other galaxies are shown in [2]).

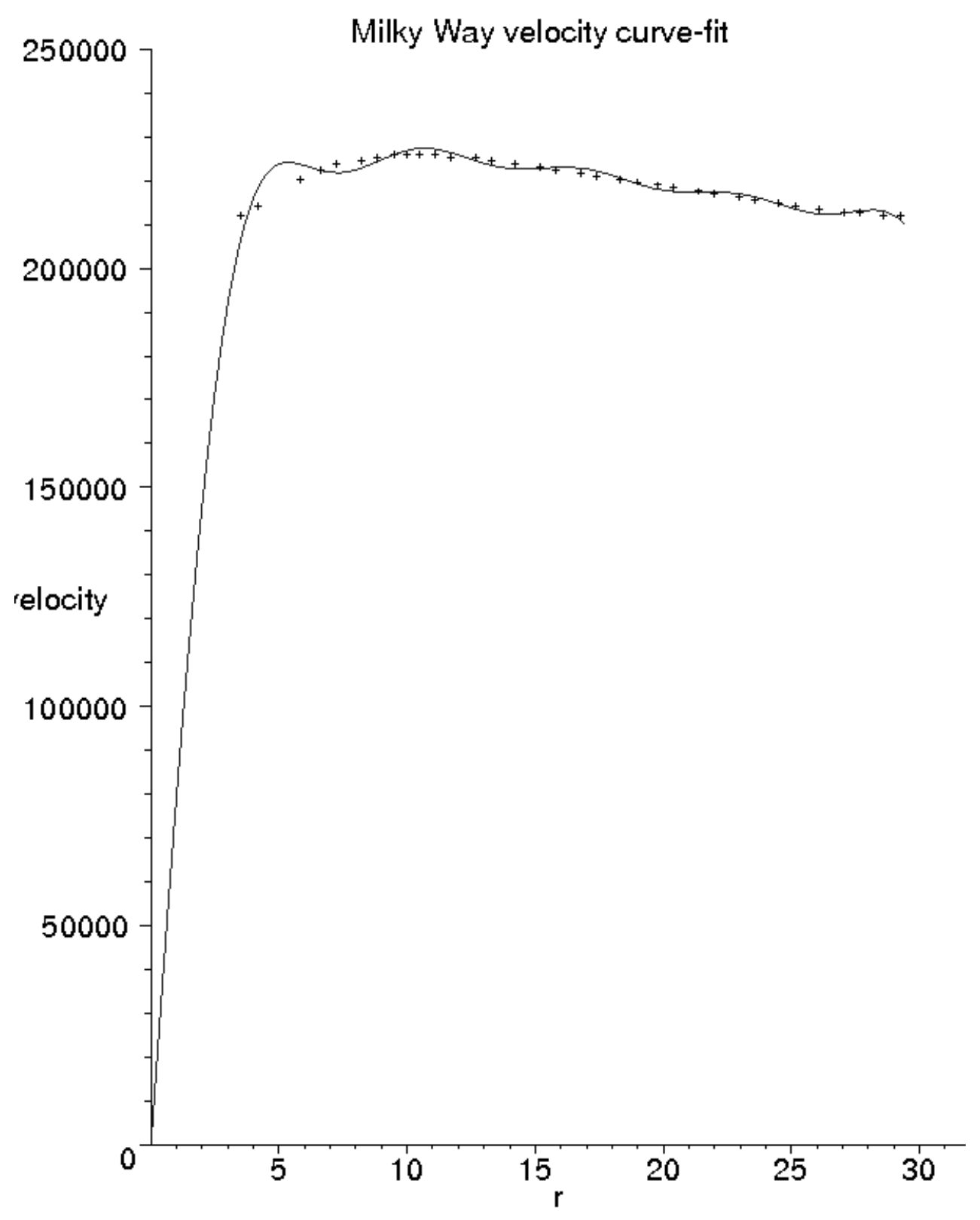

Fig. 3 - A fit of the CT model to the observed rotation curve of the Milky Way

As you can see, the CT model fits the observed rotation curve very well. However, there are some problems with the model. Most notably, the use of |z| in order to preserve symmetry produces a discontinuity in the derivative of N with respect to z at z=0. Cooperstock and Tieu claim that this has no physical consequence, mostly because the metric itself is continuous (this problem generated no less than two papers). In addition to the z=0 problem, there have been comments that uncover internal inconsistencies in the model and one that uses empirical observations about the Milky Way to call some facets of the CT model into question. To date, the concerns raised in these follow-up papers have only partially been answered by Cooperstock and Tieu.

Comments On The CT Model

Since the CT model was released, there have been no less than 5 additional papers questioning some component of the model. The first by Korzyński [4] stated that the z=0 discontinuity implied a thin, singular disk at that location. This was examined further by Vogt and Letelier [6] and they determined that the additional source of gravitational field in the form of a rotating flat disk on the galactic plane was composed of exotic, non-baryonic matter. Their conclusion was that the disk would be comprised of cosmic strings or struts with negative energy density. Also, it has been noted by Cross [1] that while the matter in the CT model does indeed rotate, the rotation is rigid and therefore cannot characterize a galaxy which is differentially rotating. This is due to the fact that the shear tensor vanishes, while the vorticity tensor has nonzero components.

Of the papers examined thus far, perhaps the most interesting is one made available on April 3rd 2006 by Burkhard Fuchs and Stefanie Phleps in which the CT model is compared to observations in the Milky Way. With empirical tests, Fuchs and Phleps [3] demonstrate that the predictions for local mass density and vertical density profile of the Milky Way are incorrect.

Burkhard and Phleps begin by arranging the formulas in the CT model as a function of density in polar coordinates:

$$\rho(r,z) = (8.36)(10^5)\left(\left(\sum_{n=1}^{10} k_n^2 C_n e^{-k_n|z|} J_0(k_n r)\right)^2 + \left(\sum_{n=1}^{10} k_n^2 C_n e^{-k_n|z|} J_1(k_n r)\right)^2 \right) \frac{M_{solar}}{pc^3}$$

At this point, they can compare the density distribution implied by the CT model with observational evidence for the Milky Way. Below is a figure displaying the predicted versus observed vertical distribution of the mass density in the Milky Way at the position of the Sun. The left panel is predicted by the CT model while the right panel is the observed distribution of stars perpendicular to the galactic midplane.

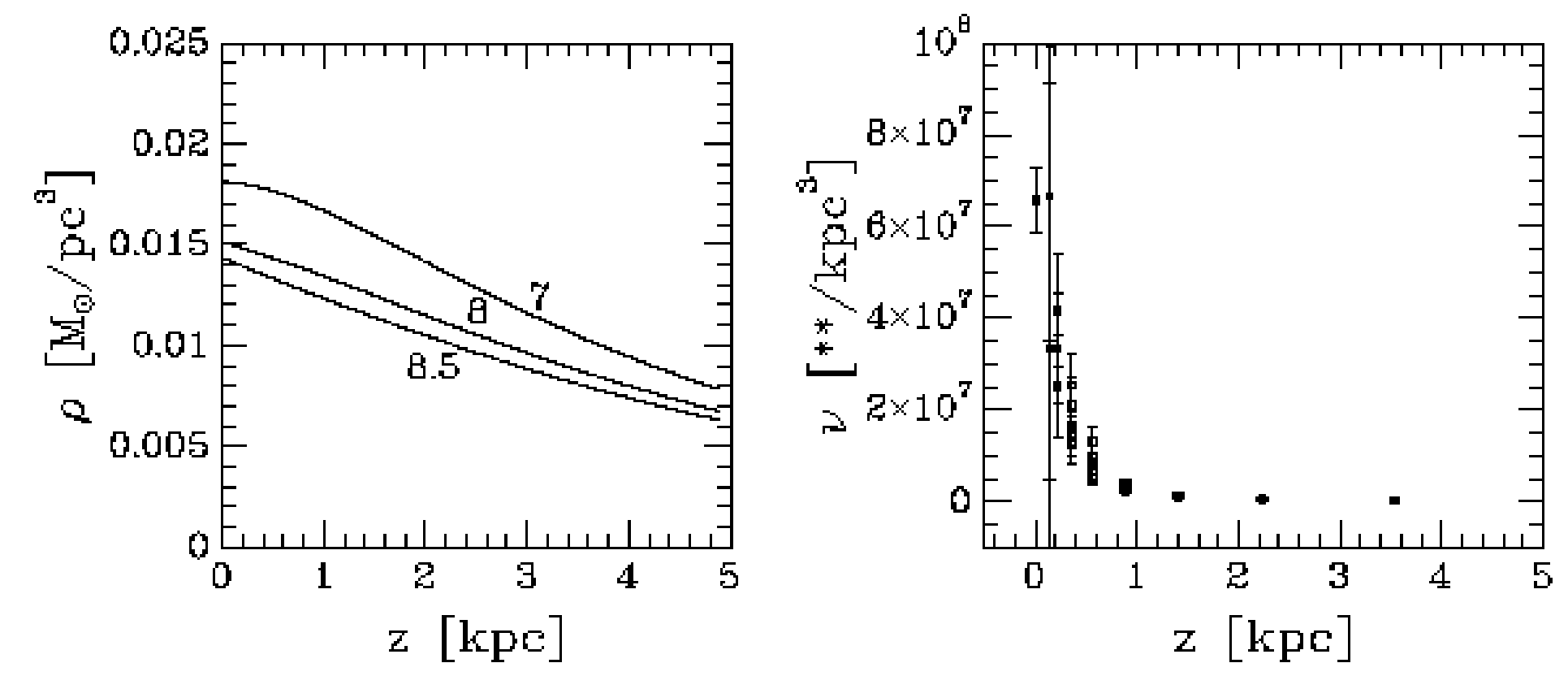

Fig. 4 – Predicted (left) versus observed (right) mass density in the Milky Way

Conclusion & Future Research

While it has been shown that the CT model is not a viable one, the necessity of dark matter and dark energy in our universe is one of, if not the, biggest open problems in astrophysics. Though we have completed our examination of the CT model, we are still working in the field of dark matter and dark energy. The 3rd year data from the WMAP probe has recently become available and has strong implications for cosmology, including a best-fit universe composed of 4% baryons, 22% dark matter and 74% dark energy. We intend to examine the cosmological implications of the 3rd year WMAP data this summer with special attention paid it's applications to the inflation that occurred just after the big bang. This work was partially supported by the United States Army Research Office (grant W911-NF-04-1-0024).

References:

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